


Benha University Faculty of Engineering- Shoubra Eng. Mathematics & Physics Department Preparatory Year		Final Term Exam Date: 20 – 12 – 2014 Course: Mathematics 1 – A Duration: 3 hours
<ul style="list-style-type: none"> <li>• Answer <b>All</b> questions</li> <li>• The Exam Consists of One page</li> </ul>	(تخلفات)	<ul style="list-style-type: none"> <li>• No. of questions: 4</li> <li>• Total Mark: 100 Marks</li> </ul>
[1] Find $y'$ from the following:		18
(a) $y = x^{\frac{2}{3}} + 2x^2$ (b) $y = \sinh x^3 \cdot \tan 3x$ (c) $y = \sin x^2 + \ln \sin 2x$ (d) $y = \cos^{-1}x^3 + \tan^{-2}x$ (e) $y^4 = x^3 + x^y$ (f) $y = t \operatorname{sect}, x = t \operatorname{cosh} t$		
[2](a) Find the following limits:		
(i) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x^2 + 16}$ (ii) $\lim_{x \rightarrow 0} \frac{\ln^4(1 + x)}{2^x - 3^x}$ (iii) $\lim_{x \rightarrow \pi} \frac{\sin 3x}{\tan 5x}$ (iv) $\lim_{x \rightarrow \infty} \frac{x^3 + 2x}{x + x^2}$		12
(b) Write the Maclurin's series of the function: $f(x) = x^2 + e^{2x}$ .		5
(c) Show that : $\sinh^{-1} x = \ln(x + \sqrt{1 + x^2})$ .		5
(d) Determine the extrema of : $f(x) = x + e^{-x}$ $g(x) = x - 2 \ln x$		10
[3] Integrate the following:		30
(a) $\int \frac{x^3}{(4 + x^2)^{3/2}} dx$ (b) $\int \frac{(x + 3)}{x^2 + 4x + 13} dx$ (c) $\int \frac{(\ln x)^3}{x} dx$ (d) $\int \cos^4 x \sin^3 x dx$ (e) $\int x^3 \sqrt{x^2 + 1} dx$ (f) $\int e^{2x} \cosh 3x dx$		
[4](a) Find the area of the surface of revaluation generated by revolving about $x$ -axis the cycloid $x = a(\theta - \sin \theta)$ , $y = (1 - \cos \theta)$ , $0 \leq \theta \leq 2\pi$		7
(b) Find the area bounded by the curves: $y = x^2$ , $x = 2$ , $x = 5$ , $y = 0$ .		7
(c) Find the volume generated by revolving, about $x$ -axis, the area bounded by: $x = a \cos \theta$ , $y = a \sin \theta$		6

*Good Luck*

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## Model Answer

### Answer Question (1)

$$[1](a) y' = \frac{2}{3}x^{-\frac{1}{3}} - 2x^2 \cdot \ln 2 \cdot 2x$$

$$(b) y' = \cosh x^3 \cdot 3x^2 \cdot \tan 3x + \sinh x^3 \cdot \sec^2 2x \cdot 2$$

$$(c) y = \cos x^2 \cdot 2x + \frac{2 \cos 2x}{\sin 2x}$$

$$(d) y' = \frac{-3x^2}{\sqrt{1-x^6}} - 2(\tan x)^{-3} \cdot \sec^2 x$$

$$(e) 4y^3 \cdot y' = 3x^2 + e^{y \ln x} (y' \ln x + \frac{y}{x})$$

$$(f) y' = \frac{\sec t + t \sec t \tan t}{\cosh t + t \sinh t}$$

### Answer Question (2)

$$(a)(i) \lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x^2+16} = \frac{2-2}{16+16} = 0$$

$$(ii) \lim_{x \rightarrow 0} \frac{[\ln(1+x)]^4}{2^x-3^x} = \frac{0}{0} = \lim_{x \rightarrow 0} \frac{\ln(1+x)}{\left(\frac{2}{3}\right)^x - 1} \frac{[\ln(1+x)]^3}{3^x} = \frac{1}{\ln 3} \cdot \frac{0}{1} = 0$$

$$(iii) \lim_{x \rightarrow \pi} \frac{\sin 3x}{\tan 5x} = \frac{0}{0} = \lim_{x \rightarrow \pi} \frac{3 \cos 3x}{5 \sec^2 5x} = \frac{3}{5}$$

$$(iv) \lim_{x \rightarrow \infty} \frac{x^2+2x}{x+x^2} = \frac{\infty}{\infty} = \lim_{x \rightarrow \infty} \frac{2x+2}{1+2x} = \frac{\infty}{\infty} = \lim_{x \rightarrow \infty} \frac{2}{2} = 1$$

(b) Since  $f(0) = 1$ ,  $f'(0) = 2$ ,  $f''(0) = 6$

Then  $f(x) = 1 + 2x + 3x^2 + \dots$

(c) If  $y = \sinh^{-1} x$ . Then  $x = \sinh y = \frac{1}{2}(e^y - e^{-y})$ . Then  $2x = e^y - e^{-y}$ .

Then  $e^{2y} - 2xe^y - 1 = 0$ . Then  $e^y = \frac{1}{2}[2x \pm \sqrt{(2x)^2 - 4(1)(-1)}] = \frac{1}{2}[2x \pm \sqrt{4x^2 + 4}]$

Then  $e^y = x \pm \sqrt{x^2 + 1}$ . Then  $y = \ln[x + \sqrt{x^2 + 1}]$

(d) From  $f'(x) = 1 - e^{-x} = 0$ , then  $x = 0$  and  $f''(x) = e^{-x}$ ,  $f''(0) = 1$

Then, the point  $x = 0$  is minimum.

From  $g'(x) = 1 - \frac{2}{x} = 0$ , then  $x = 2$  and  $g''(x) = 2x^{-2}$ ,  $g''(2) = 1$

Then, the point  $x = 2$  is minimum.

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### Answer Question (3)

(a) put  $x = 2 \tan \theta \quad \therefore dx = 2 \sec^2 \theta d\theta$ ,

$$\sqrt{4+x^2} = \sqrt{4+4 \tan^2 \theta} = 2 \sec \theta, \quad \sec \theta = \sqrt{\tan^2 \theta + 1} = \frac{1}{2} \sqrt{x^2 + 4}$$

Substitute in the problem we have

$$\begin{aligned} \therefore \int \frac{x^3}{(4+x^2)^{3/2}} dx &= \int \frac{8 \tan^3 \theta \sec^2 \theta d\theta}{8 \sec^2 \theta} = \int \frac{\tan^3 \theta d\theta}{\sec \theta} = \int \frac{\tan^2 \theta (\sec \theta \tan \theta) d\theta}{\sec^2 \theta} \\ &= \int \frac{(\sec^2 \theta - 1)(\sec \theta \tan \theta) d\theta}{\sec^2 \theta} = \int \left(1 - \frac{1}{\sec^2 \theta}\right) (\sec \theta \tan \theta) d\theta \\ &= \int \left(\sec \theta \tan \theta - \frac{\sec \theta \tan \theta}{\sec^2 \theta}\right) = \sec \theta + \frac{1}{\sec \theta} + c = \frac{1}{2} \sqrt{4+x^2} + \frac{2}{\sqrt{4+x^2}} + C \end{aligned}$$

(b)  $\int \frac{(x+3)}{x^2+4x+13} dx = \frac{1}{2} \int \frac{(2x+6)}{x^2+4x+13} dx = \frac{1}{2} \int \frac{(2x+4+2)}{x^2+4x+13} dx$

$$\begin{aligned} &= \frac{1}{2} \int \frac{2x+4}{x^2+4x+13} dx + \frac{1}{2} \int \frac{2}{x^2+4x+13} dx \\ &= \frac{1}{2} \int \frac{2x+4}{x^2+4x+13} dx + \int \frac{dx}{(x+2)^2+9} = \frac{1}{2} \ln|x^2+4x+13| + \frac{1}{3} \tan^{-1} \frac{(x+2)}{3} \end{aligned}$$

(c)  $\int \frac{(\ln x)^3}{x} dx$  Put  $u = \ln x \quad \therefore du = \frac{1}{x} dx$

$$\int \frac{(\ln x)^3}{x} dx = \int u^3 du = \frac{1}{4} u^4 + c = \frac{1}{4} (\ln u)^4 + c$$

(d)  $\int \cos^4 x \sin^3 x dx = \int \cos^4 x \sin^2 x (\sin x dx) = \int \cos^4 x (1 - \cos^2 x) (\sin x dx)$

$$= -\int (\cos^4 x - \cos^6 x) (-\sin x dx) = -\frac{1}{5} \cos^5 x - \frac{1}{7} \cos^7 x + c$$

(e)  $\int x^3 \sqrt{x^2+1} dx = \int (x^3 + x - x) \sqrt{x^2+1} dx$

$$\begin{aligned} &= \int (x^3 + x) \sqrt{x^2+1} dx - \int x \sqrt{x^2+1} dx = \int x(x^2+1) \sqrt{x^2+1} dx - \int x \sqrt{x^2+1} dx \\ &= \int x(x^2+1)^{\frac{3}{2}} dx - \int x \sqrt{x^2+1} dx = \frac{2}{10} (x^2+1)^{\frac{5}{2}} - \frac{2}{6} (x^2+1)^{\frac{3}{2}} + c \end{aligned}$$

$$(f) \int e^{2x} \cosh 3x dx = \int e^{2x} \frac{e^{3x} + e^{-3x}}{2} dx = \frac{1}{2} \int (e^{5x} + e^{-x}) dx = \left( \frac{1}{10} e^{5x} - \frac{1}{2} e^{-x} \right) + c$$


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### Answer Question (4)

(a)  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$ ,  $0 \leq \theta \leq 2\pi$

$$x = a(\theta - \sin \theta), \quad y = a(1 - \cos \theta) \quad \frac{dx}{d\theta} = a(1 - \cos \theta), \quad \frac{dy}{d\theta} = a \sin \theta$$

$$dL = \sqrt{\left[ \frac{dx}{d\theta} \right]^2 + \left[ \frac{dy}{d\theta} \right]^2} d\theta = \sqrt{a^2(1 - \cos \theta)^2 + a^2 \sin^2 \theta} = 2a \sin \frac{\theta}{2} d\theta$$

$$\begin{aligned} \therefore S_x &= 2\pi \int_a^b y \sqrt{\left[ \frac{dx}{d\theta} \right]^2 + \left[ \frac{dy}{d\theta} \right]^2} d\theta = 4\pi a \int_0^{2\pi} a(1 - \cos \theta) \sin \frac{\theta}{2} d\theta \\ &= 8\pi a^2 \int_0^{2\pi} \sin^3 \frac{\theta}{2} d\theta = 8\pi a^2 \int_0^{2\pi} \sin^2 \frac{\theta}{2} \sin \frac{\theta}{2} d\theta = 8\pi a^2 \int_0^{2\pi} (1 - \cos^2 \frac{\theta}{2}) \sin \frac{\theta}{2} d\theta \\ &= 8\pi a^2 \int_0^{2\pi} (\sin \frac{\theta}{2} - \cos^2 \frac{\theta}{2} \sin \frac{\theta}{2}) d\theta = -8\pi a^2 \left[ 2 \cos \frac{\theta}{2} + \frac{2}{3} \cos^3 \frac{\theta}{2} \right]_0^{2\pi} = \\ &= -8\pi a^2 \left[ \left( 2 \cos \frac{\pi}{2} + \frac{2}{3} \cos^3 \frac{\pi}{2} \right) - \left( 2 \cos 0 + \frac{2}{3} \cos^3 0 \right) \right] = -8\pi a^2 \left[ 0 - \left( 2 + \frac{2}{3} \right) \right] = \frac{64}{3} a^2 \pi \end{aligned}$$

(b)  $A = \int_2^5 y dx = \int_2^5 x^2 dx = \left[ \frac{1}{3} x^3 \right]_2^5 = \frac{1}{3} [5^3 - 2^3] = \frac{125 - 8}{3} = \frac{117}{3} = 39 \text{ square unit}$

(c)  $V = \pi \int_{t=0}^{t=2\pi} y^2 dx = \pi \int_{t=0}^{t=2\pi} (1 - \cos t)^2 (1 - \cos t) dt = \pi \int_{t=0}^{t=2\pi} (1 - \cos t)^3 dt$

$$\begin{aligned} &= \pi \int_{t=0}^{t=2\pi} (1 - 3\cos t + 3\cos^2 t - \cos^3 t) dt = \\ &= \pi \int_{t=0}^{t=2\pi} \left[ \frac{5}{2} - 3\cos t + \frac{3}{2} \cos 2t - (1 - \sin^2 t) \cos t \right] dt \\ &= \pi \left[ \frac{5}{2} t - 3\sin t + \frac{3}{4} \sin 2t - \left( \sin t - \frac{1}{3} \sin^3 t \right) \right]_0^{2\pi} = 5\pi^2 \end{aligned}$$


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